



Performance Evaluation of Compressive Sensing based Channel Estimation Techniques in OFDM System for Different Channels

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ABSTRACT

Data transfer in wireless communication systems requires higher data rate, transmission capability, high bandwidth and robustness. Orthogonal frequency division multiplexing (OFDM) is mainly used in regard with Multipath fading and delay. To increase the systems performance various pilot assisted method was discovered and studied. We have used compressed sensing to estimate the channel coefficients of the fading channel; then we have performed the compressive sensing (CS) recovery algorithm to estimate the channel and to nullify the fading effect, the thus much better result are obtained in the simulation which satisfies the better performance of the system as compared to the traditional method.

Keywords: Compressed sensing, channel estimation, LS, MMSE, OFDM

INTRODUCTION

The demand for flexible higher data rates has been increased due to rapid growth in technology. The frequency selective multipath fading that results in inter-symbol interference is affecting the performance of high data rates communication systems as suggested in Bajwa, Haupt, Sayeed, and Nowak (2010). In one development study (Donoho, 2006), presented an idea that OFDM is a multi-carrier modulation technique, which divides the present spectrum into some parallel subcarriers and where each subcarrier is modulated by a low rate data stream. The traditional OFDM system uses inverse fast Fourier transform (IFFT) and fast Fourier transform (FFT) for multiplexing the signals and hence removes the complexity both at transmitter and receiver. OFDM is used in areas

such as, Digital Audio and Video Broadcasting, wireless local area network (WLAN) and high performance local area network (HIPERLAN). In OFDM the original data signal will be split into multiple signals which are independent and each independent signal, is modulated at a different frequency as demonstrated by (Cheng et al., 2013).

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Channel estimation is broadly classified into two categories: blind and non-blind. The blind channel estimation requires statistical behavior of the received signals, whereas the non-blind channel estimation method requires portions of the transmitted signals as discussed by Xiong, Jiang, Gao, and You (2013). The advantage of blind channel techniques is presented by (Tropp & Gilbert, 2008) as the possible elimination of training sequences cutting the system bandwidth efficiency. In some applications training sequences need to be transmitted periodically thus causing further loss of the channel throughput. Due to these reasons blind channel estimations require a prominent concern as we have to remove the no of training sequences. On the other hand, non-blind channel estimation techniques can be implemented by giving pilot tones to all the subcarriers with a period of insertion or pilot tones into some of the carriers for each OFDM symbols. The first case is also known as block type estimation and required for slow fading channel which is further divided into LS estimation and MMSE estimation. LS estimation has less complex but its performance is not good in comparison with the MMSE estimator. From the entire estimator MMSE provides best mean-square-error (MSE) performance knowing full knowledge of channel statistics and operating signal to noise ratio (SNR) as presented by (Donoho, 2006). However if the channel statistics are not known MMSE estimator is more complex that is to be realized in the practical system. The discrete cosine transform (DCT) based channel estimation with hexagonal pilot pattern taking Doppler frequency and virtual subcarriers into considerations are shown by (Chen, Wen & Ting, 2013). The requirement of channel estimation in OFDM system to provide high data transmission and immunity to the multipath fading environment. Due to this reason, (Qi & Wu, 2011) proposed channel estimation method to provide an accurate and efficient channel to reduce the problem caused by discrete Fourier transform (DFT) transformation and to eliminate the computational complexity. An improved channel estimation method based on pilots and its performance has been checked for OFDM system. This method is very useful in mobile communication as described by (Tropp & Gilbert, 2008). The cascading approach which checked the performance of the system is presented by (Bajwa, Haupt, Raz, & Nowak, 2008). The implementation of the sparse time dispersive channel using atomic norm minimization is done having gain path and the grid of path and grid less estimation of arbitrary delay is done by (Hsieh & Wei, 2011). In the next section, we describe the various kinds of techniques in CS, channels estimation schemes, and comparison between LS, MMSE, and CS techniques as proposed by (Zhou, Tong & Zhang, 2017). The final section consists of an s of analysis of the results and the conclusion.

Model Description

We studied various estimation schemes to determine the channel coefficients and to nullify the fading components of the channel. Fourier Transform is one of the most popular transforms for obtaining frequency spectrum of signals. To find the Discrete Fourier Transform in a faster way and with reduced complexity we use Fast Fourier Transform. For the signals whose frequency does not change with time will mostly use this technique. The bit stream data that was generated will be modulated using m-ary phase shift keying (M-PSK) modulator to map input data into the symbols as described by Bajwa et al., (2010). Now, IFFT block will generate N - parallel stream using these symbols which will be transmitted over the sub carriers by OFDM as shown

in Figure 1. We can define the sequence of data as, $x(n), n = 0 \text{ to } N - 1$.

The output frequency domain $x(k)$ is defined as:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-jk\left(\frac{2\pi}{N}\right)n} \quad k = 0 \text{ to } N - 1 \quad (1)$$

The output of passing the symbols over N- Parallel streams will be determined as

$$X_k(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] e^{jk\left(\frac{2\pi}{N}\right)n} \quad (2)$$

Conventionally $X[k]$ and $X_k(n)$ are referred to as ‘Frequency domain data’ and ‘time domain data’ respectively. For simplicity, we define the FFT and IFFT when considering the various FFT algorithms as:

$$FFT_N(k, f) = \sum_{n=0}^{N-1} f(n) e^{-\frac{j2\pi kn}{N}} = \sqrt{N} F(k) \quad (3)$$

$$IFFT_N(n, F) = \sum_{k=0}^{N-1} F(k) e^{\frac{j2\pi kn}{N}} = \sqrt{N} F(k) \quad (4)$$

We consider OFDM with IFFT at transmission end and the FFT at receiver end of the system.

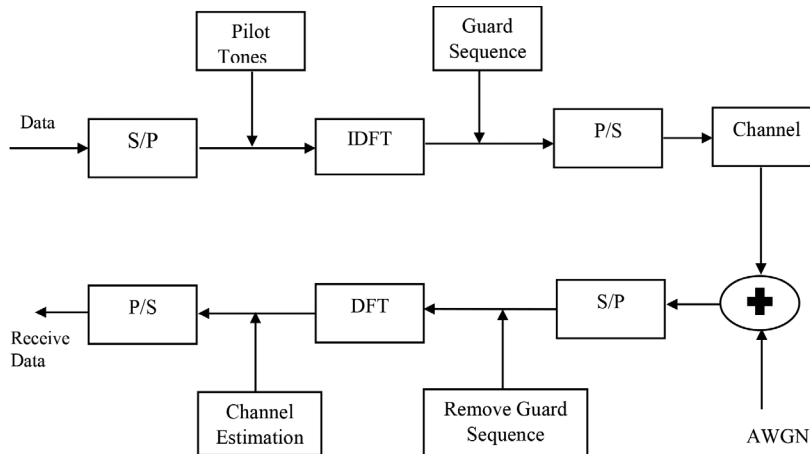


Figure 1. OFDM System model

Whereas:

R- Symbols / Sec

M – Number of data output lines in parallel

N – Samples of time domain

Lp – Prefix of length samples

Brief description is as follows:

Compress Sensing Approach. The compressed sensing is restricted to the theory that, through optimization the sparsity of a signal can be exploited using the small amount of samples required. The recovery operation is based on the two conditions. They are: sparsity in which the signal to be sparse must be in the same domain. The latter one is the incoherence that was applied through the property of isomerism must be sufficient to sparse signals. We used CS in DCT domain and to create the DCT matrix we applied the following equations:

DCT Matrix

$$y(k) = w(k) \sum_{n=1}^N x(n) \cos\left(\frac{\pi}{2N}(2n-1)(k-1)\right) \quad (5)$$

Where the values of k are from (1 to N) & for k=1, $w(k) = \frac{1}{\sqrt{N}}$, else $w(k) = \frac{\sqrt{2}}{\sqrt{N}}$

IDCT Matrix

$$y(M, l) = \sum_{k=1}^M w(k) u(k, l) \cos \pi(2M-1)(k-1) \quad (6)$$

Where for k=1, $w(k) = \frac{1}{\sqrt{M}}$, else $w(k) = \frac{2}{\sqrt{M}}$

The CS comprises of several methods that are used for the estimation. They are explained below:

Undetermined Linear Systems. These types of linear equation systems have more unknowns and have an infinite number of solutions. Therefore, there is a need of implying extra conditions or constraints on the system as presented by Cheng et al. (2013). Now, with under the determining system of linear equations we consider a sparse vector is $s_{n \times 1}$ observed in linear mixing system where $m < n$, cannot be uniquely recovered from the infinite number of solutions described by Bajwa et al. (2008).

Solution/ Reconstruction Method. The redundancy in the signals is taken as an advantage in compressed sensing as described by Hsieh and Wei (2011). In particular, many signals may have coefficients equal to zero, when represented in the same domain. By taking the weighted linear combination of samples, the compressed sensing technique will typically start as demonstrated by Xiong et al. (2013). The least-square solution to such a problem is the minimization of L^2 norm – which defines the minimum amount of energy in the system. When solving the undetermined system of linear equations, we have to minimize the number of non-zero components of the solution as presented by Tropp and Gilbert (2008). We define the function that counts the number of non-zero counts as L^0 norm. As L^0 is not computable, we will move to its closest convex L^1 as demonstrated by Chen et al., (2013). The L^1 solution is stable due to its convex nature. The basic equation can be formed as:

$$\min \|s\|_{l_1} \text{ s.t. } x = A.s \quad (7)$$

Smoothed L^0 Norm Method. As discussed earlier, we have formulated the minimization criterion for sparsity of L^0 norm as

$$\min \|s\|_{l_0} \text{ s.t. } x = A \cdot s \quad (8)$$

The L^0 form is having a disadvantage of combinatorial search and sensitivity to the noise. The basic idea of smoothed L^0 method is to the L^0 normal form with function

$$f_{\sigma}(s) \triangleq e^{-\frac{s^2}{2\sigma^2}} \quad (9)$$

Where, σ – Determines the quality of the approximation

Now we have,

$$\lim_{\sigma \rightarrow 0} f_{\sigma}(s) = \begin{cases} 1 & \text{if } s=0 \\ 0 & \text{if } s \neq 0 \end{cases} \quad (10)$$

For the vector s , we have

Where,

$$F_{\sigma}(s) = \sum_{i=1}^n f_{\sigma}(s_i) \quad (11)$$

As the values of s is non – smooth for smaller values its maximization is not global over $A \cdot s = x$. Hence, we decrease the sequence of σ which gives the minimum L^0 norm solution.

Least Square estimation. OFDM channel estimation symbols will be transferred periodically in block-type pilot-based channel estimation. In this type based on the pilot signals and received signals estimating the channel conditions will be done. The receiver uses the estimated channel condition to decode the data inside the block. The least-square estimator minimizes the parameter:

$$(\bar{Y} - \underline{X}\bar{H})^H (\bar{Y} - \underline{X}\bar{H}) \quad (12)$$

Where,

$(.)^H$ – Conjugate transpose operation

$\bar{Y}$ – received signals

$\underline{X}$ Pilot signal specified by matrix

Then, the LS estimator of $\bar{H}$ is given by

$$\hat{H}_{LS} = \underline{X}^{-1} \bar{Y} = [(X_k/Y_k)]^T \quad (k=0, \dots, N-1) \quad (13)$$

The LS estimator can calculate the channel conditions even without having any knowledge about the statistics of the channel with very low complexity for calculation. But these calculations are suffering from high mean square error.

MMSE Estimation. The MMSE method minimizes the mean square error by employing second – order statistics of the channel conditions as presented by Bajwa et al. (2008). Assuming that the channel vector $\bar{g}$ and the noise $\bar{N}$ are uncorrelated, it is derived that:

$$R_{HH} = E\{H^H H\} = F R_g g F^H \quad (14)$$

$$R_{gY} = E\{\bar{g} \bar{Y}^H\} = R_{gg, F^H X H} \quad (15)$$

$$R_{YY} = E\{\bar{Y} \bar{Y}^H\} \quad (16)$$

$$= X F R_g g F^H X H + \sigma^2 N I_N \quad (17)$$

R_{aa}, R_{HH}, R_{YY} – Represents auto-covariance matrix of $\bar{g}, \bar{H}, \bar{Y}$ respectively

R_{gY} – Represents the cross-covariance matrix between $\bar{g}$ and $\bar{Y}$

σ_N^2 – Represents the noise variance

By assuming that the auto-covariance matrix and noise variance are known at the receiver in advance, and the MMSE estimator is of $\bar{g}$ is given by $\hat{g}_{MMSE}$ may not be a minimum mean square error. At last, it is calculated that:

$$\hat{g}_{mmse} = R_{gY}^{-1} Y^{HH} \quad (18)$$

$$\hat{H}_{mmse} = F \hat{g}_{mmse} \quad (19)$$

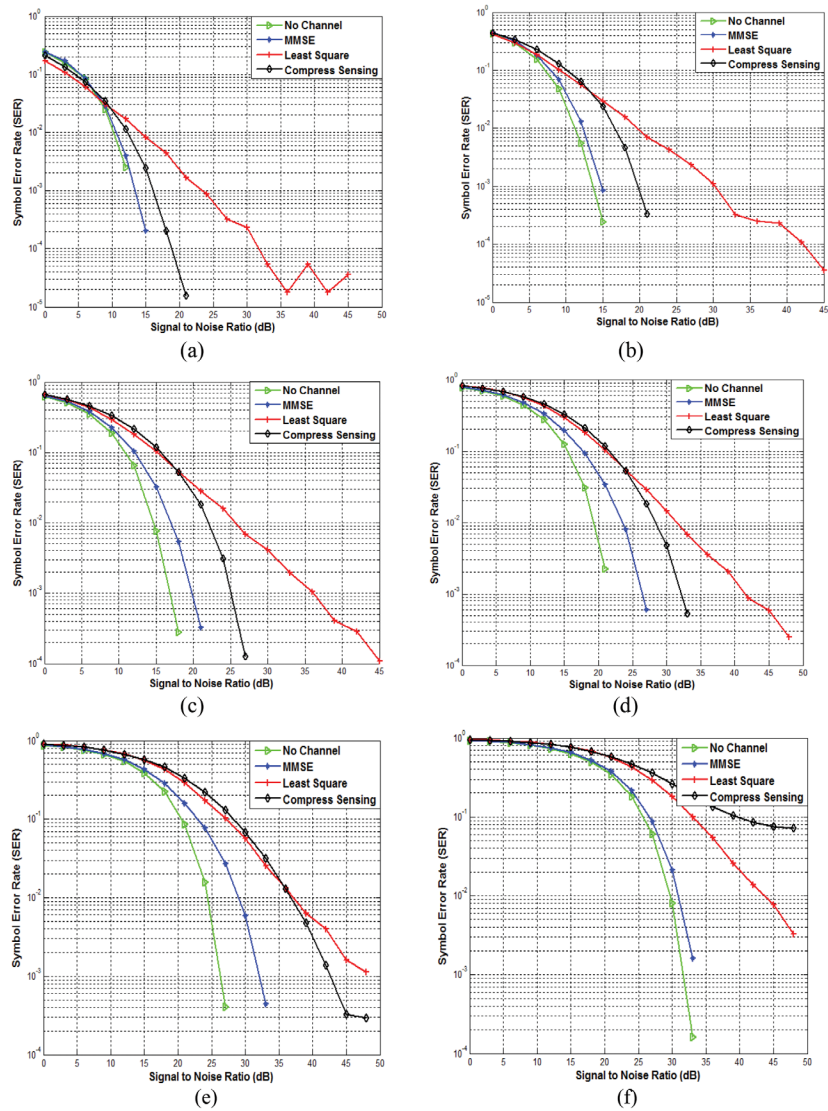
The MMSE estimator surely yields much better performance than the LS estimator, but the only major drawback is its high computational complexity.

RESULTS AND DISCUSSION

In this section, we have depicted the compressed sensing techniques using DCT transform and evaluated its performance in different channel models, comparing it with traditional techniques like LS, MMSE and with no channel. The comparison is shown in Table 1. We have given the simulation results to compare the performance of the proposed LS, CS and MMSE techniques both at Rayleigh and Rician channel for OFDM system. The data sequences are modulated by M-PSK modulation. The compressed sensing techniques use DCT transform for compressing the data thus, performance is evaluated at particular channel respectively. Here all these estimation schemes are done over Rayleigh channel thus for each modulation schemes graph has been shown below from Figure 2 (a-h). At higher modulation, it becomes difficult to recover the signal from sparse property thus the performance of the CS based estimator deteriorates this is shown in Figure 2(f-h). We observe that SNR required to achieve the symbol error rate (SER) up to 10^{-3} for different estimation schemes at Rayleigh channel are: For LS estimation, it takes about 23dB while for MMSE it takes only 14dB. CS require 16dB and for no channel condition, it takes only 13dB to achieve the same for binary phase shift keying (BPSK) modulation. Similarly, we observed for quadrature phase shift keying (QPSK) modulation that SNR required for LS estimation is 30dB, MMSE is 15dB while for no channel it is 14dB and CS required 20dB to achieve that SER. Similarly, this is also observed for higher order modulation to validate the estimators' performance.

Table 1
BER performance in Rayleigh & Rician Channel

M-PSK	SNR VALUE TO ACHIEVE BER UPTO 10 ⁻³ (dB)							
	RAYLEIGH Channel				RICIAN Channel			
	NO Channel	LS	MMSE	CS	NO Channel	LS	MMSE	CS
2	13	23	14	16	13	25	35	16
4	14	30	15	20	14	32	18	20
8	17	35	20	25	18	37	27	25
16	22	40	27	32	22	42	32	33
32	26	45	32	42	27	46	28	42
64	32	>45	35	>45	32	48	41	>45
128	37	>45	>45	>45	37	>45	40	>45
256	42	>45	>45	>45	43	>45	45	>45



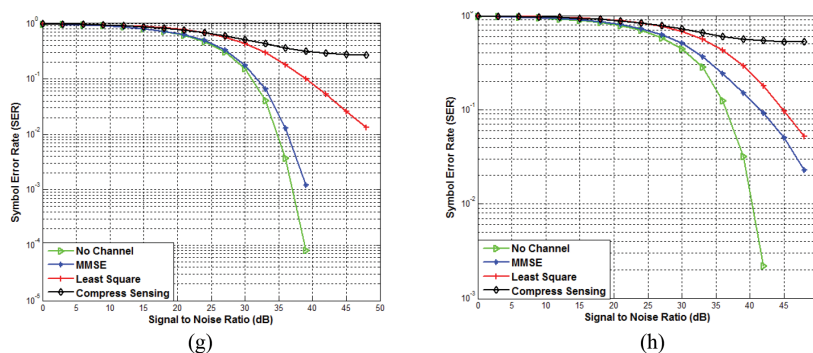
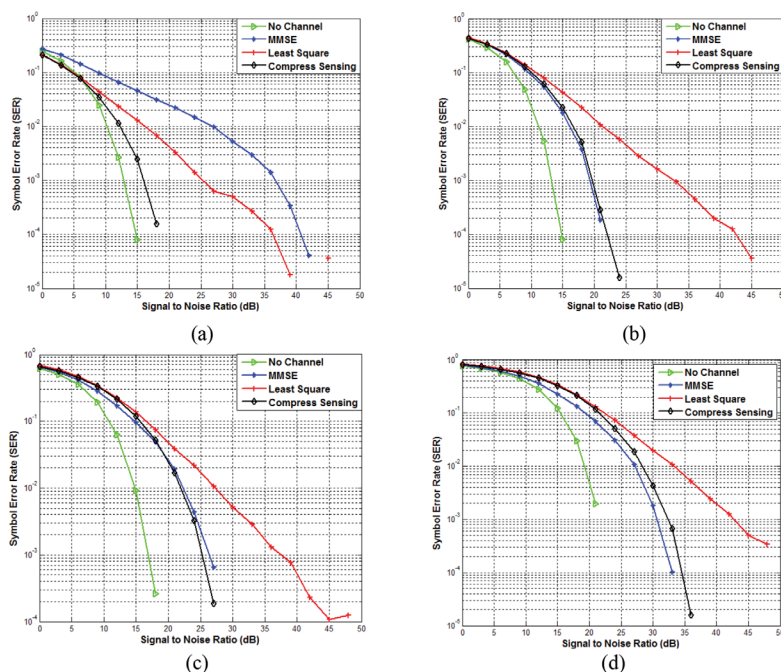


Figure 2 (a-h). SER & SNR comparison for LS, MMSE & CS-DCT estimating technique in Rayleigh Channel (a) BPSK (b) QPSK (c) 8-PSK (d) 16-PSK (e) 32-PSK (f) 64-PSK (g) 128-PSK (h) 256-PSK

Figure 3(a-h). shows the performance of the estimation schemes over Rician channel at which CS based estimation shows a better response at low SNR and at high SNR it matches to LS estimation and its performance deteriorates. In rician channel also, the performance of the LS estimation Scheme is worst. MMSE being the best and CS shows a better response at low SNR as SNR is increased cs performance matches to LS estimation. At higher modulation, it's become difficult to recover the signal from sparse property thus the performance of the CS based estimator deteriorates this is shown in Figure 3(f-h). We observe that SNR required to achieve the SER up to 10^{-3} for different estimation schemes at Rician channel are: For LS estimation, it takes about 25dB while for MMSE it takes only 35dB. CS require 16dB and for no channel condition, it takes only 13dB to achieve the same for BPSK modulation. Similarly, we observed for QPSK modulation that SNR required for LS estimation is 32dB, MMSE is 18dB while for no channel it is 14dB and CS required 20dB to achieve that SER. Similarly, this is also observed for higher order modulation to validate the estimators' performance.



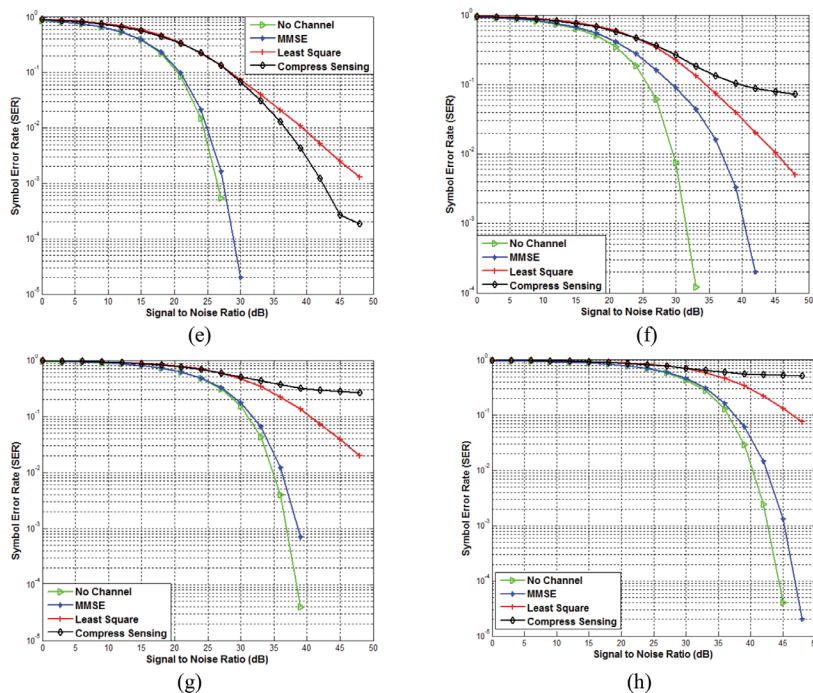


Figure 3 (a-h). SER & SNR comparison for LS, MMSE & CS-DCT estimating technique in Rician Channel (a) BPSK (b) QPSK (c) 8-PSK (d) 16-PSK (e) 32-PSK (f) 64-PSK (g) 128-PSK (h) 256-PSK

CONCLUSION

In this paper the channel estimation schemes based on compressed sensing theory was studied. We first showed the performance evaluation of LS, MMSE and CS method based on M-PSK modulation. Based on channel statistics we made a comparison of the listed schemes on different channel and plot has been made between SNR & SER to show the estimator performance. Through simulation, we concluded that LS estimation was the worst as it required much SNR to achieve the fixer error rate while MMSE was the best. But practically knowing channel state information is not possible so implemented a CS based approach which depicts that at the low value of SNR CS gives the best response but as SNR increases the performance of the CS based estimation deteriorates as proved in higher modulation types. The estimation schemes can be tested for other digital modulation schemes.

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